# AP Calculus BC Summer Packet

This summer packet will be due on the first day of the 2018-2019 school year. Limits, derivatives, and integrals are the 3 main topics you are expected to know and will be assessed on the third class meeting. To be successful in this course a strong foundation in Calculus AB is needed. We will use quarter 1 to review all of the content from AB Calculus. If you have any questions feel free to email me at <u>guerrerali@watertownps.org</u>

The pages you will need are attached. Please do all work on a separate sheet of paper and be sure to label your assignments.

1.) Limits: pg118 (11-24, 39-41, 69-74, 81-86)

2.) Derivatives: pg204 ( 11-26, 31-42, 43, 45, 49,51, 57-77, 99-102)

3.) Integration: pg362 (1-7, 27-29, 37-42, 47, 53-56, 65, 67, 73)

## 1 Review Exercises

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

**Precalculus or Calculus** **In Exercises 1 and 2, determine whether the problem can be solved using precalculus or whether calculus is required. If the problem can be solved using precalculus, solve it. If the problem seems to require calculus, explain your reasoning and use a graphical or numerical approach to estimate the solution.**

1. Find the distance between the points (1, 1) and (3, 9) along the curve $y = x^2$.
2. Find the distance between the points (1, 1) and (3, 9) along the line $y = 4x - 3$.

**Estimating a Limit Numerically** **In Exercises 3 and 4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result.**

3. $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-7x+12}$

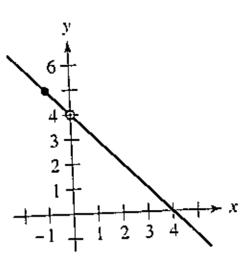
| $x$ | 2.9 | 2.99 | 2.999 | 3 | 3.001 | 3.01 | 3.1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | ? | | | |

4. $\lim\limits_{x\to 0} \dfrac{\sqrt{x+4}-2}{x}$

| $x$ | $-0.1$ | $-0.01$ | $-0.001$ | 0 | 0.001 | 0.01 | 0.1 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | ? | | | |

**Finding a Limit Graphically** **In Exercises 5 and 6, use the graph to find the limit (if it exists). If the limit does not exist, explain why.**

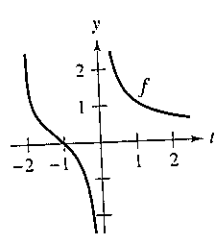
5. $h(x) = \dfrac{4x - x^2}{x}$




(a) $\lim\limits_{x\to 0} h(x)$

(b) $\lim\limits_{x\to -1} h(x)$

6. $f(t) = \dfrac{\ln(t+2)}{t}$




(a) $\lim\limits_{t\to 0} f(t)$

(b) $\lim\limits_{t\to -1} f(t)$

**Using the $\varepsilon$–$\delta$ Definition of a Limit** **In Exercises 7–10, find the limit $L$. Then use the $\varepsilon$–$\delta$ definition to prove that the limit is $L$.**

7. $\lim\limits_{x\to 1} (x+4)$
8. $\lim\limits_{x\to 9} \sqrt{x}$
9. $\lim\limits_{x\to 2} (1-x^2)$
10. $\lim\limits_{x\to 5} 9$

**Finding a Limit** **In Exercises 11–28, find the limit.**

11. $\lim\limits_{x\to -6} x^2$
12. $\lim\limits_{x\to 0} (5x-3)$
13. $\lim\limits_{x\to 6} (x-2)^2$
14. $\lim\limits_{x\to -5} \sqrt[3]{x-3}$
15. $\lim\limits_{x\to 4} \dfrac{4}{x-1}$
16. $\lim\limits_{x\to 2} \dfrac{x}{x^2+1}$
17. $\lim\limits_{t\to -2} \dfrac{t+2}{t^2-4}$
18. $\lim\limits_{t\to 4} \dfrac{t^2-16}{t-4}$
19. $\lim\limits_{x\to 4} \dfrac{\sqrt{x-3}-1}{x-4}$
20. $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x}$
21. $\lim\limits_{x\to 0} \dfrac{[1/(x+1)]-1}{x}$
22. $\lim\limits_{s\to 0} \dfrac{(1/\sqrt{1+s})-1}{s}$
23. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{\sin x}$
24. $\lim\limits_{x\to \pi/4} \dfrac{4x}{\tan x}$
25. $\lim\limits_{x\to 1} e^{x-1} \sin \dfrac{\pi x}{2}$
26. $\lim\limits_{x\to 2} \dfrac{\ln(x-1)^2}{\ln(x-1)}$
27. $\lim\limits_{\Delta x\to 0} \dfrac{\sin[(\pi/6)+\Delta x]-(1/2)}{\Delta x}$

[*Hint:* $\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$]

28. $\lim\limits_{\Delta x\to 0} \dfrac{\cos(\pi+\Delta x)+1}{\Delta x}$

[*Hint:* $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$]

**Evaluating a Limit** **In Exercises 29–32, evaluate the limit given $\lim\limits_{x\to c} f(x) = -6$ and $\lim\limits_{x\to c} g(x) = \frac{1}{2}$.**

29. $\lim\limits_{x\to c} [f(x)g(x)]$
30. $\lim\limits_{x\to c} \dfrac{f(x)}{g(x)}$
31. $\lim\limits_{x\to c} [f(x)+2g(x)]$
32. $\lim\limits_{x\to c} [f(x)]^2$

**Graphical, Numerical, and Analytic Analysis** **In Exercises 33–36, use a graphing utility to graph the function and estimate the limit. Use a table to reinforce your conclusion. Then find the limit by analytic methods.**

33. $\lim\limits_{x\to 0} \dfrac{\sqrt{2x+9}-3}{x}$
34. $\lim\limits_{x\to 0} \dfrac{[1/(x+4)]-(1/4)}{x}$
35. $\lim\limits_{x\to 0} \dfrac{20(e^{x/2}-1)}{x-1}$
36. $\lim\limits_{x\to 0} \dfrac{\ln(x+1)}{x+1}$

**Free-Falling Object** **In Exercises 37 and 38, use the position function $s(t) = -4.9t^2 + 250$, which gives the height (in meters) of an object that has fallen for $t$ seconds from a height of 250 meters. The velocity at time $t = a$ seconds is given by**

$$\lim_{t\to a} \frac{s(a)-s(t)}{a-t}.$$

37. Find the velocity of the object when $t = 4$.
38. At what velocity will the object impact the ground?

**ing a Limit** **In Exercises 39–46, find the limit (if it s). If it does not exist, explain why.**

9. $\lim\limits_{x\to3^+} \dfrac{1}{x+3}$

40. $\lim\limits_{x\to6^-} \dfrac{x-6}{x^2-36}$

1. $\lim\limits_{x\to4^-} \dfrac{\sqrt{x}-2}{x-4}$

42. $\lim\limits_{x\to3^-} \dfrac{|x-3|}{x-3}$

3. $\lim\limits_{x\to2^-} (2[\![x]\!]+1)$

44. $\lim\limits_{x\to4} [\![x-1]\!]$

5. $\lim\limits_{x\to2} f(x)$, where $f(x) = \begin{cases} (x-2)^2, & x \le 2 \\ 2-x, & x > 2 \end{cases}$

46. $\lim\limits_{s\to-2} f(s)$, where $f(s) = \begin{cases} -s^2-4s-2, & s \le -2 \\ s^2+4s+6, & s > -2 \end{cases}$

**Removable and Nonremovable Discontinuities** **In Exercises 47–50, find the $x$-values (if any) at which $f$ is not continuous. Which of the discontinuities are removable?**

47. $f(x) = x^2 - 4$

48. $f(x) = \dfrac{1}{x^2-9}$

49. $f(x) = \dfrac{x}{x^3-x}$

50. $f(x) = \dfrac{x+3}{x^2-3x-18}$

**Testing for Continuity** **In Exercises 51–58, describe the intervals on which the function is continuous.**

51. $f(x) = -3x^2 + 7$

52. $f(x) = \dfrac{4x^2+7x-2}{x+2}$

53. $f(x) = \sqrt{x-4}$

54. $f(x) = [\![x+3]\!]$

55. $g(x) = 2e^{[\![x]\!]/4}$

56. $h(x) = -2\ln|5-x|$

57. $f(x) = \begin{cases} \dfrac{3x^2-x-2}{x-1}, & x \ne 1 \\ 0, & x = 1 \end{cases}$

58. $f(x) = \begin{cases} 5-x, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$

**Using the Intermediate Value Theorem** **In Exercises 59 and 60, use the Intermediate Value Theorem to show that the function has a zero in the given interval.**

59. $f(x) = 2x^3 - 3$; $[1, 2]$

60. $f(x) = 2\ln(x+4) - 4$; $[3, 4]$

61. **Finding Limits** Let $f(x) = (x^2-4)/|x-2|$. Find each limit (if it exists).
    (a) $\lim\limits_{x\to2^-} f(x)$ (b) $\lim\limits_{x\to2^+} f(x)$ (c) $\lim\limits_{x\to2} f(x)$

62. **Finding Limits** For $f(x) = \sqrt{x(x-1)}$, find (a) the domain of $f$, (b) $\lim\limits_{x\to0^-} f(x)$, and (c) $\lim\limits_{x\to1^+} f(x)$.

**Finding Vertical Asymptotes** **In Exercises 63–68, find the vertical asymptotes (if any) of the graph of the function.**

63. $f(x) = \dfrac{x^3}{x^2-9}$

64. $h(x) = \dfrac{6x}{36-x^2}$

65. $g(x) = \dfrac{2x+1}{x^2-64}$

66. $f(x) = \csc \pi x$

67. $g(x) = \ln(25 - x^2)$

68. $f(x) = 7e^{-3/x}$

**Finding a One-Sided Limit** **In Exercises 69–78, find the one-sided limit (if it exists).**

69. $\lim\limits_{x\to1^-} \dfrac{x^2+2x+1}{x-1}$

70. $\lim\limits_{x\to(1/2)^+} \dfrac{x}{2x-1}$

71. $\lim\limits_{x\to-1^+} \dfrac{x+1}{x^3+1}$

72. $\lim\limits_{x\to-1^-} \dfrac{x+1}{x^4-1}$

73. $\lim\limits_{x\to0^-} \left(x - \dfrac{1}{x^3}\right)$

74. $\lim\limits_{x\to2^-} \dfrac{1}{\sqrt[3]{x^2-4}}$

75. $\lim\limits_{x\to0^+} \dfrac{\sin 4x}{5x}$

76. $\lim\limits_{x\to0^+} \dfrac{\sec x}{x}$

77. $\lim\limits_{x\to0^-} \ln(\sin x)$

78. $\lim\limits_{x\to0^-} 12e^{-2/x}$

79. **Environment** A utility company burns coal to generate electricity. The cost $C$ in dollars of removing $p\%$ of the air pollutants in the stack emissions is
    $$C = \frac{80{,}000p}{100-p}, \quad 0 \le p < 100.$$
    (a) Find the cost of removing 15%, 50%, and 90% of the pollutants.
    (b) Find the limit of $C$ as $p$ approaches 100 from the left and interpret its meaning.

80. **Limits and Continuity** The function $f$ is defined as
    $$f(x) = \frac{\tan 2x}{x}, \quad x \ne 0.$$
    (a) Find $\lim\limits_{x\to0} \dfrac{\tan 2x}{x}$ (if it exists).
    (b) Can the function $f$ be defined at $x = 0$ such that it is continuous at $x = 0$?

**Finding a Limit** **In Exercises 81–88, find the limit.**

81. $\lim\limits_{x\to\infty} \left(8 + \dfrac{1}{x}\right)$

82. $\lim\limits_{x\to\infty} \dfrac{1-4x}{x+1}$

83. $\lim\limits_{x\to\infty} \dfrac{2x^2}{3x^2+5}$

84. $\lim\limits_{x\to\infty} \dfrac{4x^2}{x^4+3}$

85. $\lim\limits_{x\to-\infty} \dfrac{3x^2}{x+5}$

86. $\lim\limits_{x\to-\infty} \dfrac{\sqrt{x^2+x}}{-2x}$

87. $\lim\limits_{x\to-\infty} \dfrac{6x}{x+\cos x}$

88. $\lim\limits_{x\to-\infty} \dfrac{x}{2\sin x}$

**Horizontal Asymptotes** **In Exercises 89–94, use a graphing utility to graph the function and identify any horizontal asymptotes.**

89. $f(x) = \dfrac{3}{x} - 2$

90. $g(x) = \dfrac{5x^2}{x^2+2}$

91. $h(x) = \dfrac{2x+3}{x-4}$

92. $f(x) = \dfrac{3x}{\sqrt{x^2+2}}$

93. $f(x) = \dfrac{5}{3+2e^{-x}}$

94. $h(x) = 10\ln\dfrac{x}{x+1}$

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## 2 Review Exercises

**Finding the Derivative by the Limit Process** **In Exercises 1–4, find the derivative of the function by the limit process.**

1. $f(x) = 12$
2. $f(x) = 5x - 4$
3. $f(x) = x^2 - 4x + 5$
4. $f(x) = \dfrac{6}{x}$

**Using the Alternative Form of the Derivative** **In Exercises 5 and 6, use the alternative form of the derivative to find the derivative at $x = c$ (if it exists).**

5. $g(x) = 2x^2 - 3x, \quad c = 2$
6. $f(x) = \dfrac{1}{x + 4}, \quad c = 3$

**Determining Differentiability** **In Exercises 7–10, describe the $x$-values at which $f$ is differentiable.**

7. $f(x) = \dfrac{2}{x - 3}$
8. $f(x) = \dfrac{3x}{x + 1}$
9. $f(x) = \sqrt{x - 1}$
10. $f(x) = (x - 3)^{2/5}$

**Finding a Derivative** **In Exercises 11–22, use the rules of differentiation to find the derivative of the function.**

11. $y = 25$
12. $f(t) = 4t^4$
13. $f(x) = x^3 - 11x^2$
14. $g(s) = 3s^5 - 2s^4$
15. $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$
16. $f(x) = x^{1/2} - x^{-1/2}$
17. $g(t) = \dfrac{2}{3t^2}$
18. $h(x) = \dfrac{8}{5x^4}$
19. $f(\theta) = 4\theta - 5 \sin \theta$
20. $g(\alpha) = 4 \cos \alpha + 6$
21. $f(t) = 3 \cos t - 4e^t$
22. $g(s) = \frac{5}{3} \sin s - 2e^s$

**Finding the Slope of a Graph** **In Exercises 23–26, find the slope of the graph of the functions at the given point.**

23. $f(x) = \dfrac{27}{x^3}, \ (3, 1)$
24. $f(x) = 3x^2 - 4x, \ (1, -1)$
25. $f(x) = 2x^4 - 8, \ (0, -8)$
26. $f(\theta) = 3 \cos \theta - 2\theta, \ (0, 3)$

27. **Vibrating String** When a guitar string is plucked, it vibrates with a frequency of $F = 200\sqrt{T}$, where $F$ is measured in vibrations per second and the tension $T$ is measured in pounds. Find the rates of change of $F$ when (a) $T = 4$ and (b) $T = 9$.

28. **Volume** The surface area of a cube with sides of length $s$ is given by $S = 6s^2$. Find the rates of change of the surface area with respect to $s$ when (a) $s = 3$ inches and (b) $s = 5$ inches.

**Vertical Motion** **In Exercises 29 and 30, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.**

29. A ball is thrown straight down from the top of a 600-foot building with an initial velocity of $-30$ feet per second.
    (a) Determine the position and velocity functions for the ball.
    (b) Determine the average velocity on the interval $[1, 3]$.
    (c) Find the instantaneous velocities when $t = 1$ and $t = 3$.
    (d) Find the time required for the ball to reach ground level.
    (e) Find the velocity of the ball at impact.

30. To estimate the height of a building, a weight is dropped from the top of the building into a pool at ground level. The splash is seen 9.2 seconds after the weight is dropped. What is the height (in feet) of the building?

**Finding a Derivative** **In Exercises 31–42, find the derivative of the function.**

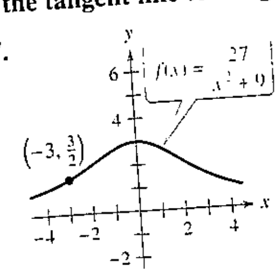
31. $f(x) = (5x^2 + 8)(x^2 - 4x - 6)$
32. $g(x) = (2x^3 + 5x)(3x - 4)$
33. $h(x) = \sqrt{x} \sin x$
34. $f(t) = 2t^5 \cos t$
35. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$
36. $f(x) = \dfrac{2x + 7}{x^2 + 4}$
37. $y = \dfrac{x^4}{\cos x}$
38. $y = \dfrac{\sin x}{x^4}$
39. $y = 3x^2 \sec x$
40. $y = 2x - x^2 \tan x$
41. $y = 4xe^x - \cot x$
42. $g(x) = 3x \sin x + x^2 \cos x$

**Finding an Equation of a Tangent Line** **In Exercises 43–46, find an equation of the tangent line to the graph of $f$ at the given point. Use the *tangent* feature of a graphing utility to confirm your results.**

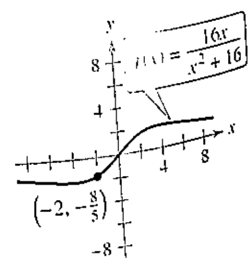
43. $f(x) = (x + 2)(x^2 + 5), \ (-1, 6)$
44. $f(x) = (x - 4)(x^2 + 6x - 1), \ (0, 4)$
45. $f(x) = \dfrac{x + 1}{x - 1}, \ \left(\dfrac{1}{2}, -3\right)$
46. $f(x) = \dfrac{1 + \cos x}{1 - \cos x}, \ \left(\dfrac{\pi}{2}, 1\right)$

**Famous Curves** **In Exercises 47 and 48, find an equation of the tangent line to the graph at the given point.**

47.




48.



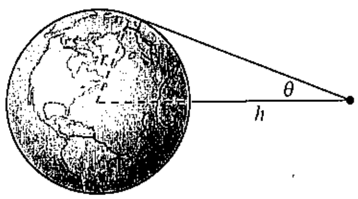
**...ing a Second Derivative** **In Exercises 49–54, find ...econd derivative of the function.**

...$(t) = -8t^3 - 5t + 12$ **50.** $h(x) = 6x^{-2} + 7x^2$

...$(x) = 15x^{5/2}$ **52.** $f(x) = 20\sqrt[5]{x}$

...$(\theta) = 3 \tan \theta$ **54.** $h(t) = 10 \cos t - 15 \sin t$

...**Satellites** When satellites observe Earth, they can scan only part of Earth's surface. Some satellites have sensors that can measure the angle $\theta$ shown in the figure. Let $h$ represent the satellite's distance from Earth's surface, and let $r$ represent Earth's radius.




(a) Show that $h = r(\csc \theta - 1)$.

(b) Find the rate at which $h$ is changing with respect to $\theta$ when $\theta = 30°$. (Assume $r = 3960$ miles.)

**56. Acceleration** The velocity of an automobile starting from rest is

$$v(t) = \frac{90t}{4t + 10}$$

where $v$ is measured in feet per second. Find the acceleration at (a) 1 second, (b) 5 seconds, and (c) 10 seconds.

**Finding a Derivative** **In Exercises 57–82, find the derivative of the function.**

**57.** $y = (7x + 3)^4$ **58.** $y = (x^2 - 6)^3$

**59.** $y = \dfrac{1}{x^2 + 4}$ **60.** $f(x) = \dfrac{1}{(5x + 1)^2}$

**61.** $y = 5 \cos(9x + 1)$

**62.** $y = 1 - \cos 2x + 2 \cos^2 x$

**63.** $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$ **64.** $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

**65.** $y = x(6x + 1)^5$

**66.** $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

**67.** $f(x) = \dfrac{3x}{\sqrt{x^2 + 1}}$ **68.** $h(x) = \left(\dfrac{x + 5}{x^2 + 3}\right)^2$

**69.** $g(t) = t^2 e^{t/4}$ **70.** $h(z) = e^{-z^2/2}$

**71.** $y = \sqrt{e^{2x} + e^{-2x}}$ **72.** $y = 3e^{-3/t}$

**73.** $g(x) = \dfrac{x^2}{e^x}$ **74.** $f(\theta) = \dfrac{1}{2} e^{\sin 2\theta}$

**75.** $g(x) = \ln \sqrt{x}$ **76.** $h(x) = \ln \dfrac{x(x - 1)}{x - 2}$

**77.** $f(x) = x\sqrt{\ln x}$ **78.** $f(x) = \ln[x(x^2 - 2)^{2/3}]$

**79.** $y = \dfrac{1}{b^2}\left[\ln(a + bx) + \dfrac{a}{a + bx}\right]$

**80.** $y = \dfrac{1}{b^2}[a + bx - a \ln(a + bx)]$

**81.** $y = -\dfrac{1}{a} \ln \dfrac{a + bx}{x}$

**82.** $y = -\dfrac{1}{ax} + \dfrac{b}{a^2} \ln \dfrac{a + bx}{x}$

**Evaluating a Derivative** **In Exercises 83–88, find and evaluate the derivative of the function at the given point.**

**83.** $f(x) = \sqrt{1 - x^3}$, $(-2, 3)$

**84.** $f(x) = \sqrt[3]{x^2 - 1}$, $(3, 2)$

**85.** $f(x) = \dfrac{4}{x^2 + 1}$, $(-1, 2)$

**86.** $f(x) = \dfrac{3x + 1}{4x - 3}$, $(4, 1)$

**87.** $y = \dfrac{1}{2} \csc 2x$, $\left(\dfrac{\pi}{4}, \dfrac{1}{2}\right)$

**88.** $y = \csc 3x + \cot 3x$, $\left(\dfrac{\pi}{6}, 1\right)$

**Finding a Second Derivative** **In Exercises 89–92, find the second derivative of the function.**

**89.** $y = (8x + 5)^3$ **90.** $y = \dfrac{1}{5x + 1}$

**91.** $f(x) = \cot x$ **92.** $y = \sin^2 x$

**93. Refrigeration** The temperature $T$ (in degrees Fahrenheit) of food in a freezer is

$$T = \frac{700}{t^2 + 4t + 10}$$

where $t$ is the time in hours. Find the rate of change of $T$ with respect to $t$ at each of the following times. Interpret the results in the context of the problem.

(a) $t = 1$ (b) $t = 3$ (c) $t = 5$ (d) $t = 10$

**94. Harmonic Motion** The displacement from equilibrium of an object in harmonic motion on the end of a spring is

$$y = \frac{1}{4} \cos 8t - \frac{1}{4} \sin 8t$$

where $y$ is measured in feet and $t$ is the time in seconds. Determine the position and velocity of the object when $t = \pi/4$.

**95. Circulatory System** The speed $S$ of blood that is $r$ centimeters from the center of an artery is

$$S = C(R^2 - r^2)$$

where $C$ is a constant, $R$ is the radius of the artery, and $S$ is measured in centimeters per second. After a drug is administered, the artery begins to dilate at a rate of $dR/dt$. At a constant distance $r$, find the rate at which $S$ changes with respect to $t$ for $C = 1.76 \times 10^5$, $R = 1.2 \times 10^{-2}$, and $dR/dt = 10^{-5}$.

**96. Modeling Data** The atmospheric pressure decreases with increasing altitude. At sea level, the average air pressure is one atmosphere (1.033227 kilograms per square centimeter). The table gives the pressures $p$ (in atmospheres) at various altitudes $h$ (in kilometers).

| $h$ | 0 | 5 | 10 | 15 | 20 | 25 |
|---|---|---|---|---|---|---|
| $p$ | 1 | 0.55 | 0.25 | 0.12 | 0.06 | 0.02 |

(a) Use a graphing utility to find a model of the form $p = a + b \ln h$ for the data. Explain why the result is an error message.

(b) Use a graphing utility to find the logarithmic model $h = a + b \ln p$ for the data.

(c) Use a graphing utility to plot the data and graph the logarithmic model.

(d) Use the model to estimate the altitude at which the pressure is 0.75 atmosphere.

(e) Use the model to estimate the pressure at an altitude of 13 kilometers.

(f) Find the rates of change of pressure when $h = 5$ and $h = 20$. Interpret the results in the context of the problem.

**97. Modeling Data** The normal daily maximum temperatures $T$ (in degrees Fahrenheit) for Chicago, Illinois, are shown in the table. *(Source: National Oceanic and Atmospheric Administration)*

| Month | Jan | Feb | Mar | Apr |
|---|---|---|---|---|
| Temperature | 29.6 | 34.7 | 46.1 | 58.0 |

| Month | May | Jun | Jul | Aug |
|---|---|---|---|---|
| Temperature | 69.9 | 79.2 | 83.5 | 81.2 |

| Month | Sep | Oct | Nov | Dec |
|---|---|---|---|---|
| Temperature | 73.9 | 62.1 | 47.1 | 34.4 |

(a) Use a graphing utility to plot the data and find a model for the data of the form

$$T(t) = a + b \sin(ct - d)$$

where $T$ is the temperature and $t$ is the time in months, with $t = 1$ corresponding to January.

(b) Use a graphing utility to graph the model. How well does the model fit the data?

(c) Find $T'$ and use a graphing utility to graph the derivative.

(d) Based on the graph of the derivative, during what times does the temperature change most rapidly? Most slowly? Do your answers agree with your observations of the temperature changes? Explain.

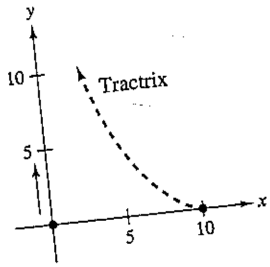
**98. Tractrix** A person walking along a dock drags a boat by a 10-meter rope. The boat travels along a path known as a *tractrix* (see figure). The equation of this path is

$$y = 10 \ln\left(\frac{10 + \sqrt{100 - x^2}}{x}\right) - \sqrt{100 - x^2}.$$

(a) Use a graphing utility to graph the function.

(b) What is the slope of the path when $x = 5$ and $x = 9$?

(c) What does the slope of the path approach as $x \to 10$?




**Finding a Derivative** **In Exercises 99–104, find $dy/dx$ by implicit differentiation.**

99. $x^2 + y^2 = 64$

100. $x^2 + 4xy - y^3 = 6$

101. $x^3y - xy^3 = 4$

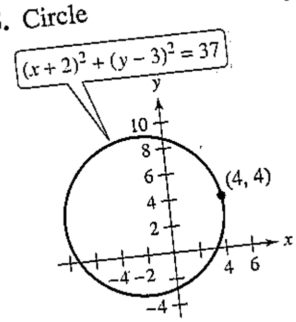
102. $\sqrt{xy} = x - 4y$

103. $x \sin y = y \cos x$

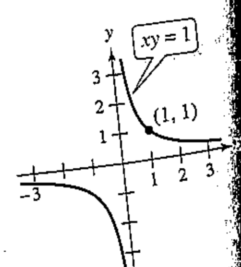
104. $\cos(x + y) = x$

**Famous Curves** **In Exercises 105–108, find an equation of the tangent line to the graph at the given point. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

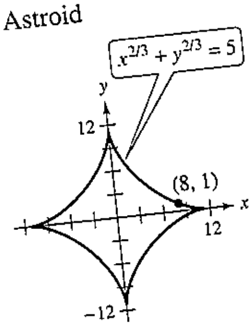
105. Circle



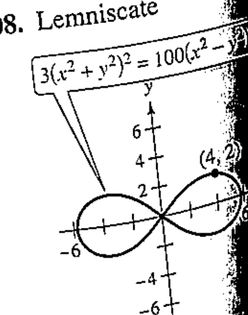

106. Rotated hyperbola




107. Astroid




108. Lemniscate



## 4 Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Finding an Indefinite Integral** In Exercises 1–6, find the indefinite integral.

1. $\int (4x^2 + x + 3)\, dx$
2. $\int \frac{6}{\sqrt[3]{x}}\, dx$
3. $\int \frac{x^4 + 8}{x^3}\, dx$
4. $\int (5 \cos x - 2 \sec^2 x)\, dx$
5. $\int (5 - e^x)\, dx$
6. $\int \frac{10}{x}\, dx$

**Finding a Particular Solution** In Exercises 7–10, find the particular solution that satisfies the differential equation and the initial condition.

7. $f'(x) = -6x,\ f(1) = -2$
8. $f'(x) = 9x^2 + 1,\ f(0) = 7$
9. $f''(x) = 24x,\ f'(-1) = 7,\ f(1) = -4$
10. $f''(x) = 2 \cos x,\ f'(0) = 4,\ f(0) = -5$

11. **Velocity and Acceleration** A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second. Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)
    (a) How long will it take the ball to rise to its maximum height? What is the maximum height?
    (b) After how many seconds is the velocity of the ball one-half the initial velocity?
    (c) What is the height of the ball when its velocity is one-half the initial velocity?

12. **Velocity and Acceleration** The speed of a car traveling in a straight line is reduced from 45 to 30 miles per hour in a distance of 264 feet. Find the distance in which the car can be brought to rest from 30 miles per hour, assuming the same constant deceleration.

**Finding a Sum** In Exercises 13 and 14, find the sum. Use the summation capabilities of a graphing utility to verify your result.

13. $\sum_{i=1}^{5} (5i - 3)$

14. $\sum_{k=0}^{3} (k^2 + 1)$

**Using Sigma Notation** In Exercises 15 and 16, use sigma notation to write the sum.

15. $\frac{1}{3(1)} + \frac{1}{3(2)} + \frac{1}{3(3)} + \cdots + \frac{1}{3(10)}$

16. $\left(\frac{3}{n}\right)\left(\frac{1+1}{n}\right)^2 + \left(\frac{3}{n}\right)\left(\frac{2+1}{n}\right)^2 + \cdots + \left(\frac{3}{n}\right)\left(\frac{n+1}{n}\right)^2$

**Evaluating a Sum** In Exercises 17–20, use the properties of summation and Theorem 4.2 to evaluate the sum.

17. $\sum_{i=1}^{20} 2i$
18. $\sum_{i=1}^{30} (3i - 4)$
19. $\sum_{i=1}^{20} (i + 1)^2$
20. $\sum_{i=1}^{12} i(i^2 - 1)$

**Finding Area by the Limit Definition** In Exercises 21–24, use the limit process to find the area of the region bounded by the graph of the function and the $x$-axis over the given interval. Sketch the region.

21. $y = 8 - 2x,\ [0, 3]$
22. $y = x^2 + 3,\ [0, 2]$
23. $y = 5 - x^2,\ [-2, 1]$
24. $y = \frac{1}{4}x^3,\ [2, 4]$

25. **Finding Area by the Limit Definition** Use the limit process to find the area of the region bounded by $x = 5y - y^2$, $x = 0$, $y = 2$, and $y = 5$.

26. **Upper and Lower Sums** Consider the region bounded by $y = mx$, $y = 0$, $x = 0$, and $x = b$.
    (a) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/4$.
    (b) Find the upper and lower sums to approximate the area of the region when $\Delta x = b/n$.
    (c) Find the area of the region by letting $n$ approach infinity in both sums in part (b). Show that, in each case, you obtain the formula for the area of a triangle.

**Evaluating a Definite Integral Using a Geometric Formula** In Exercises 27 and 28, sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral.

27. $\int_0^5 (5 - |x - 5|)\, dx$
28. $\int_{-6}^{6} \sqrt{36 - x^2}\, dx$

29. **Using Properties of Definite Integrals** Given $\int_4^8 f(x)\, dx = 12$ and $\int_4^8 g(x)\, dx = 5$, evaluate
    (a) $\int_4^8 [f(x) + g(x)]\, dx$.
    (b) $\int_4^8 [f(x) - g(x)]$
    (c) $\int_4^8 [2f(x) - 3g(x)]\, dx$.
    (d) $\int_4^8 7f(x)\, dx$.

30. **Using Properties of Definite Integrals** $\int_0^3 f(x)\, dx = 4$ and $\int_3^6 f(x)\, dx = -1$, evaluate
    (a) $\int_0^6 f(x)\, dx$.
    (b) $\int_6^3 f(x)\, dx$.
    (c) $\int_4^4 f(x)\, dx$.
    (d) $\int_3^6 -10f(x)$

grating a Function with a Discontinuity In cises 31 and 32, (a) find the $x$-value(s) at which the tion is not continuous, and (b) evaluate the integral.

$\int_{-6}^{8} \frac{x^2 - 25}{5 + x}\, dx$

$\int_{3/2}^{6} h(x)\, dx$, where $h(x) = \begin{cases} 4, & x < 4 \\ \frac{3}{4}x - 2, & x \geq 4 \end{cases}$

ing the Trapezoidal Rule In Exercises 33–36, pproximate the definite integral using the Trapezoidal ule with $n = 4$. Compare the result with the pproximation of the integral using a graphing utility.

3. $\int_{2}^{3} \frac{2}{1 + x^2}\, dx$ 34. $\int_{0}^{1} \frac{x^{3/2}}{3 - x^2}\, dx$

35. $\int_{0}^{3} \sqrt{x} \ln(x + 1)\, dx$ 36. $\int_{0}^{\pi} \sqrt{1 + \sin^2 x}\, dx$

Evaluating a Definite Integral In Exercises 37–42, use the Fundamental Theorem of Calculus to evaluate the definite integral.

37. $\int_{0}^{8} (3 + x)\, dx$ 38. $\int_{2}^{3} (x^4 + 4x - 6)\, dx$

39. $\int_{4}^{9} x\sqrt{x}\, dx$ 40. $\int_{-\pi/4}^{\pi/4} \sec^2 t\, dt$

41. $\int_{0}^{2} (x + e^x)\, dx$ 42. $\int_{1}^{6} \frac{3}{x}\, dx$

Finding the Area of a Region In Exercises 43–46, find the area of the region bounded by the graphs of the equations.

43. $y = 8 - x,\ x = 0,\ x = 6,\ y = 0$

44. $y = \sqrt{x}(1 - x),\ y = 0$

45. $y = \frac{2}{x},\ y = 0,\ x = 1,\ x = 3$

46. $y = 1 + e^x,\ y = 0,\ x = 0,\ x = 2$

Finding the Average Value of a Function In Exercises 47 and 48, find the average value of the function over the given interval and all values of $x$ in the interval for which the function equals its average value.

47. $f(x) = \frac{1}{\sqrt{x}},\ [4, 9]$ 48. $f(x) = x^3,\ [0, 2]$

Using the Second Fundamental Theorem of Calculus In Exercises 49–52, use the Second Fundamental Theorem of Calculus to find $F'(x)$.

49. $F(x) = \int_{0}^{x} t^2\sqrt{1 + t^3}\, dt$ 50. $F(x) = \int_{1}^{x} \frac{1}{t^2}\, dt$

51. $F(x) = \int_{-3}^{x} (t^2 + 3t + 2)\, dt$

52. $F(x) = \int_{0}^{x} \csc^2 t\, dt$

Finding an Indefinite Integral In Exercises 53–64, find the indefinite integral.

53. $\int \frac{x^2}{\sqrt{x^3 + 3}}\, dx$ 54. $\int 6x^3\sqrt{3x^4 + 2}\, dx$

55. $\int x(1 - 3x^2)^4\, dx$ 56. $\int \frac{x + 4}{(x^2 + 8x - 7)^2}\, dx$

57. $\int \sin^3 x \cos x\, dx$ 58. $\int x \sin 3x^2\, dx$

59. $\int \frac{\cos \theta}{\sqrt{1 - \sin \theta}}\, d\theta$ 60. $\int \frac{e^{1/x}}{x^2}\, dx$

61. $\int (x + 1)5^{(x+1)^2}\, dx$ 62. $\int \frac{1}{t^2}(2^{-1/t})\, dt$

63. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x\, dx$

64. $\int \sec 2x \tan 2x\, dx$

Evaluating a Definite Integral In Exercises 65–72, evaluate the definite integral. Use a graphing utility to verify your result.

65. $\int_{0}^{1} (3x + 1)^5\, dx$ 66. $\int_{0}^{1} x^2(x^3 - 2)^3\, dx$

67. $\int_{0}^{3} \frac{1}{\sqrt{1 + x}}\, dx$ 68. $\int_{3}^{6} \frac{x}{3\sqrt{x^2 - 8}}\, dx$

69. $2\pi \int_{0}^{1} (y + 1)\sqrt{1 - y}\, dy$ 70. $2\pi \int_{-1}^{0} x^2\sqrt{x + 1}\, dx$

71. $\int_{0}^{\pi} \cos \frac{x}{2}\, dx$ 72. $\int_{-\pi/4}^{\pi/4} \sin 2x\, dx$

Finding an Indefinite Integral In Exercises 73–76, find the indefinite integral.

73. $\int \frac{1}{7x - 2}\, dx$ 74. $\int \frac{x^2}{x^3 + 1}\, dx$

75. $\int \frac{\sin x}{1 + \cos x}\, dx$ 76. $\int \frac{e^{2x}}{e^{2x} + 1}\, dx$

Evaluating a Definite Integral In Exercises 77–80, evaluate the definite integral.

77. $\int_{1}^{4} \frac{2x + 1}{2x}\, dx$ 78. $\int_{1}^{e} \frac{\ln x}{x}\, dx$

79. $\int_{0}^{\pi/3} \sec \theta\, d\theta$ 80. $\int_{0}^{\pi} \tan \frac{\theta}{3}\, d\theta$

Finding an Indefinite Integral In Exercises 81–86, find the indefinite integral.

81. $\int \frac{1}{e^{2x} + e^{-2x}}\, dx$ 82. $\int \frac{1}{3 + 25x^2}\, dx$

83. $\int \frac{x}{\sqrt{1 - x^4}}\, dx$ 84. $\int \frac{1}{x\sqrt{9x^2 - 49}}\, dx$

85. $\int \frac{\arctan (x/2)}{4 + x^2}\, dx$ 86. $\int \frac{\arcsin 2x}{\sqrt{1 - 4x^2}}\, dx$